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Josephson junction microcalorimeter with a superconductor loop

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Abstract

We propose a new microcalorimeter in which the critical current of a Josephson junction can be varied by an electron temperature in the normal metal barrier of the superconductor–normal metal–superconductor (SNS) or superconductor–normal metal–insulator–superconductor (SNIS) junctions. In this detector, a Josephson junction with a radiation absorber is included in a superconductor loop and the change of its critical current is converted into a change of magnetic flux in the loop. We estimated the energy resolution of this detector by calculating a noise equivalent power (NEP) of the detector. The estimated energy resolution and dynamic range are 4.2 eV/5.8 eV and 3.1 keV/6.2 keV, respectively with an Ag absorber of $500 \times 500 \times 2\ \mu m^3$ at 100 mK.

1. Introduction

Radiation detectors with high energy resolution are required in various fields: x-ray astronomy, dark matter search, neutrino spectrometry and material analysis. Cryogenic radiation spectrometer devices have been under development in a number of countries for many years. Currently, the best energy resolution for x-ray photons has been obtained by semiconductor microcalorimeters in which neutron transmutation doped (NTD) germanium thermistors are coupled to superconductor absorbers [1] and a superconductor transition edge sensor (TES) whose resistance varies abruptly around T_c [2].

In this paper, we propose a new microcalorimeter: a Josephson junction microcalorimeter with a superconductor loop. In this microcalorimeter, magnetic flux in the superconductor loop with a Josephson junction is kept constant by feedback method in a modulation of the critical current of the Josephson junction. The energy deposited by radiation raises electron temperature in a radiation absorber. The temperature change, converted to the change of the critical current of the Josephson junction, is measured as a feedback signal.

Superconductor–insulator–superconductor (SIS) tunnel junction detectors and normal metal-insulator-superconductor (NIS) tunnel junction detectors have already been developed as cryogenic radiation detectors with superconductor junctions. In SIS tunnel junction detectors, quasiparticles are made in superconductor absorbers by energy deposition of incident radiation particles, and the number of quasiparticles is detected as a tunnel current. Josephson current, which is not utilized but is treated as a noise source, must be suppressed by external magnetic field in SIS tunnel junction detectors. So, SIS tunnel junction detectors are completely different from a Josephson junction microcalorimeter with a superconductor loop depending on whether Josephson current is used or not. A NIS tunnel junction detector, in which the subgap current varies exponentially with electron temperature of the normal metal, is similar to a Josephson junction microcalorimeter in that the change of electron temperature of the normal metal is utilized. A NIS tunnel junction detector has a shot noise of tunnel current, which causes both thermal and current noise. So, the energy resolution of a NIS tunnel junction detector is not as good as that of a Josephson junction microcalorimeter.

2. Detection principle

2.1. SNS and SNIS Josephson junctions

In this microcalorimeter, a superconductor–normal metal–superconductor (SNS) or a superconductor–normal metal–insulator–superconductor (SNIS) junction is used as a Josephson junction. Critical currents of SNS and SNIS junctions depend on the electron temperature in their normal metal barriers.

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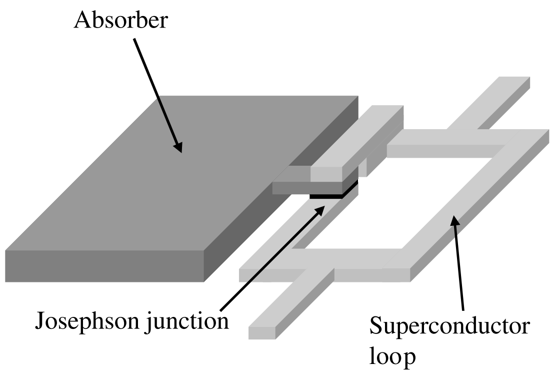

Figure 1. Schematic diagram of the proposed Josephson junction microcalorimeter.

When an operating temperature is well below the critical temperature of the superconductor of an SNS junction, the critical current of an SNS junction, I_1, is written as [4]

$$I_1(T) = A\exp\left(-\frac{2x}{\xi_n}\right) = A\exp\left[-\frac{2x}{\sqrt{\hbar D_n/2\pi kT}}\right], \quad (1)$$

where A is the proportional constant, x is the thickness of a normal metal barrier, $\xi_n = \sqrt{\hbar D_n/2\pi kT}$ is the coherence length in the normal metal, D_n is the diffusion coefficient in the normal metal, k is the Boltzmann constant and T is the electron temperature in the normal metal. In equation (1), the dirty limit $l_n \ll \xi_n$ is assumed, where l_n is the mean free path of electrons in the normal metal, and the motion of superconducting electrons is well described as a diffusion process. In the case of an SNIS junction, critical current is proportional to the product of the probability amplitudes of superconducting electrons in both sides of the insulator layer. At the SN side of the SNIS junction, a probability amplitude is obtained by considering the diffusion of superconducting electrons from the superconductor layer into the normal metal layer. So, the temperature dependence of the critical current of an SNIS junction is also described approximately by equation (1) [5].

Equation (1) indicates that the critical current of an SNS or SNIS junction changes exponentially as a function of the square root of the electron temperature in a normal metal barrier. By coupling a radiation absorber to the normal metal barrier of a Josephson junction, the temperature change due to the radiation incidence is converted to the change of the critical current of the Josephson junction. To measure the change of the critical current of the Josephson junction efficiently, we suggest a device in which the Josephson junction is included in a superconductor loop. In this device, a change of the critical current is converted to the change of magnetic flux in the superconductor loop.

The proposed device structure is illustrated schematically in figure 1. A radiation absorber is connected to the normal metal barrier of a Josephson junction through a bridge. When the bridge is made of Cu, thermal conductivity and heat capacity at 100 mK are 0.73 (J m^{-1} s^{-1} K^{-1}) and 9.8 (J K^{-1} m^{-3}), respectively. Then, the temperature diffusion coefficient is 0.075 (m^2 s^{-1}), and the heat transfer time is in the order of 10 ns, in case the length of the bridge is 30 μm. In this condition, a radiation absorber and the normal metal barrier of a Josephson junction can be assumed at the same temperature.

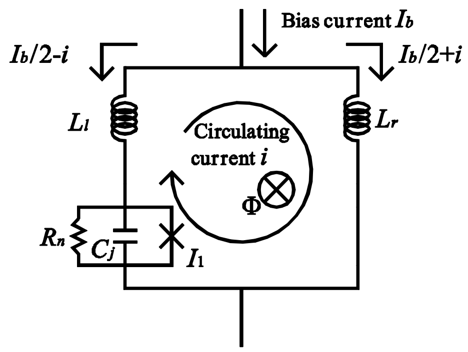

Figure 2. A simple model circuit of a superconductor loop with a Josephson junction described in RCSJ model.

2.2. Superconductor loop with a Josephson junction

We investigate magnetic flux in a superconductor loop with a Josephson junction (SLJJ). A simple model circuit of an SLJJ is shown in figure 2. For simplicity, a Josephson junction is described in a model with the resistively and capacitively shunted junction (RCSJ). In figure 2, R_n, C_j and I_1 are the normal state resistance, the capacitance and the critical current of the Josephson junction, respectively.

Inductance of a superconductor loop is divided into L_l and L_r. A bias current I_b is split into $I_b/2 - i$ and $I_b/2 + i$, where i is the circulating current. Then, a magnetic flux

$$\Phi = Li + I_b(L_r - L_l)/2 \quad (2)$$

is generated in the loop. $L = L_l + L_r$ is the inductance of the whole loop. For simplicity, the condition $L_r = L_l$ is assumed.

A relation between the gauge invariant phase difference across the junction, ψ, and magnetic flux in the loop, Φ, is given by

$$\psi = 2\pi\Phi/\Phi_0 - 2n\pi, \quad (3)$$

where $\Phi_0 = 2.07 \times 10^{-15}$ (Wb) is the magnetic flux quantum. Current across the junction is given by

$$\frac{1}{2}I_b - i = \frac{1}{R_n}V + I_1\sin\psi + \frac{d}{dt}\left(C_j V\right), \quad (4)$$

where V is the voltage across the junction and is written as

$$V = \frac{d\Phi}{dt}. \quad (5)$$

Substitution of equations (2), (3) and (5) into equation (4) gives the following equation for magnetic flux Φ,

$$\frac{1}{R_n}\frac{d\Phi}{dt} + C_j\frac{d^2\Phi}{dt^2} = -\frac{\Phi}{L} - I_1\sin 2\pi\frac{\Phi}{\Phi_0} + \frac{I_b}{2} = -\frac{d}{d\Phi}U(\Phi), \quad (6)$$

where the potential energy $U(\Phi)$ is defined as

$$U(\Phi) = \frac{1}{2}\frac{\Phi^2}{L} - I_1\frac{\Phi_0}{2\pi}\cos 2\pi\frac{\Phi}{\Phi_0} - \frac{1}{2}I_b\Phi. \quad (7)$$

In equation (7), the first and the third term are energies of the circulating current and the bias current respectively, and the second term is the energy of the Josephson junction.

Stable states are realized when equation (6) is 0 and $\frac{d^2}{d\Phi^2}U(\Phi) > 0$. With the introduction of a variable $2\pi\frac{\Phi}{\Phi_0} = \phi$ for convenience in calculation, we obtain the equation

$$I_1\sin\phi = \frac{1}{2}I_b - \frac{\Phi_0}{2\pi L}\phi. \quad (8)$$

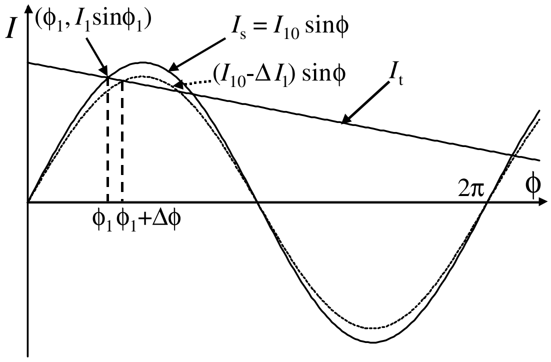

Figure 3. A total current I_t and a superconducting current I_s flow in the junction side of the loop. The intersection point of an I_s–ϕ curve and I_t–ϕ line changes according to the critical current of the Josephson junction.

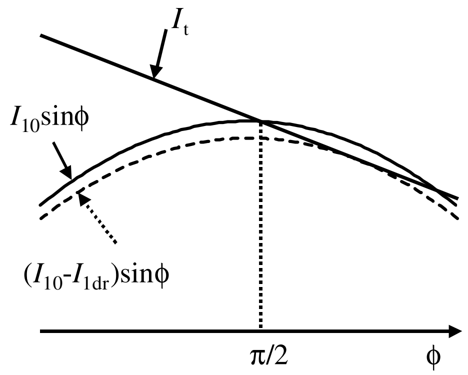

Figure 4. Dynamic range of the critical current I_1. When $I_1 = I_{10} - I_{1dr}$, the I_s–ϕ curve is tangent to the I_t–ϕ line. A jump of magnetic flux to the next stable state occurs when I_1 decreases further.

The left-hand side and the right-hand side of equation (8) indicate superconducting current I_s and total current I_t across the Josephson junction, respectively. Thus, at a stable state, all current across the Josephson junction is superconducting current and the voltage across the junction is zero. Equation (8) also indicates that ϕ depends on I_1.

2.3. Sensitivity and dynamic range of a Josephson junction microcalorimeter

In having a solution of equation (8), we employ a graph as shown in figure 3. In figure 3, a superconducting current

$$I_s = I_1 \sin\phi, \tag{9}$$

and total current

$$I_t = \frac{1}{2}I_b - \frac{\Phi_0}{2\pi L}\phi = I_{10}\left\{\sin\phi_1 + \frac{1}{2\pi a}(\phi_1 - \phi)\right\} \tag{10}$$

are drawn, where $a = LI_{10}/\Phi_0$, I_{10} is the critical current of a Josephson junction at a base temperature and ϕ_1 is the ϕ coordinate of an intersection point of the I_s–ϕ curve and the I_t–ϕ line at a base temperature. To investigate the sensitivity of the Josephson junction microcalorimeter to the critical current, we differentiate the equation $I_s = I_t$ by I_1. After some calculation, the following relationship

$$\frac{\partial\phi}{\partial I_1} = \frac{1}{I_{10}}\frac{\sin^2\phi}{\cos\phi\left\{\sin\phi_1 + \frac{1}{2\pi a}(\phi_1 - \phi)\right\} + \frac{1}{2\pi a}\sin\phi} \tag{11}$$

is obtained. The sensitivity of the microcalorimeter is given by

$$\left.\frac{\partial\Phi}{\partial I_1}\right|_{\Phi=\Phi_0(\frac{\phi_1}{2\pi})} = L\frac{\sin\phi_1}{2\pi a\cos\phi_1 + 1}. \tag{12}$$

The sensitivity at $\phi_1 = \pi/2$ is L, which indicates that the change of the critical current equals that of the circulating current at $\phi_1 = \pi/2$. The sensitivity diverges when $\cos\phi_1 = -\frac{1}{2\pi a}$. At this point, the I_s–ϕ curve is tangent to the I_t–ϕ line as shown in figure 4 and a jump of magnetic flux to the next stable state occurs when the critical current I_1 decreases further; in other words, the dynamic range of I_1 is zero. The dynamic range of I_1 is defined as the current range from I_{10} to the one at which the jump of magnetic flux occurs. As ϕ_1 approaches $\phi_{1c} = \arccos\left(-\frac{1}{2\pi a}\right)$, the sensitivity becomes larger and the dynamic range becomes smaller. The dynamic range of I_1 at $\phi_1 = \pi/2$, I_{1dr}, is approximately calculated to be

$$I_{1dr} = I_{10}\frac{1}{8\pi a^2}. \tag{13}$$

In calculating equation (13), we assumed that the point of tangency is close to $\pi/2$ and the I_s–ϕ curve can be approximated to a parabola around $\pi/2$.

3. Energy resolution

In this section, we calculate the energy resolution of the Josephson junction microcalorimeter. We consider two noise contributions: (1) a noise from a fluctuation of critical current I_1 according to the temperature fluctuation of an absorber and (2) a noise from a Johnson noise of the normal state resistance in SLJJ.

3.1. Fluctuation of a critical current

Magnetic flux fluctuation due to the critical current of a Josephson junction is written as

$$(\delta\Phi_I)^2 = \left(\frac{\partial\Phi}{\partial I_1}\right)^2 (\delta I_1)^2. \tag{14}$$

The fluctuation of the critical current of a Josephson junction, δI_1, is caused by a fluctuation of a temperature of a normal metal absorber, δT, and written as

$$(\delta I_1)^2 = \left(\frac{dI_1}{dT}\right)^2 (\delta T)^2. \tag{15}$$

$\left(\frac{dI_1}{dT}\right)$ is obtained by differentiating equation (1) by T as,

$$\frac{dI_1}{dT} = I_1(T)\frac{-x}{\sqrt{\hbar D_n/2\pi k}}T^{-1/2}. \tag{16}$$

The power flow equation for a radiation absorber is given as [6]

$$C\frac{dT}{dt} = -g(T - T_0) + P(t), \tag{17}$$

where C is the heat capacity of an electron system in the absorber, g is the heat conductance between an electron system and a phonon system in a normal metal absorber, T_0 is the base temperature and $P(t)$ is the power flow into a normal metal

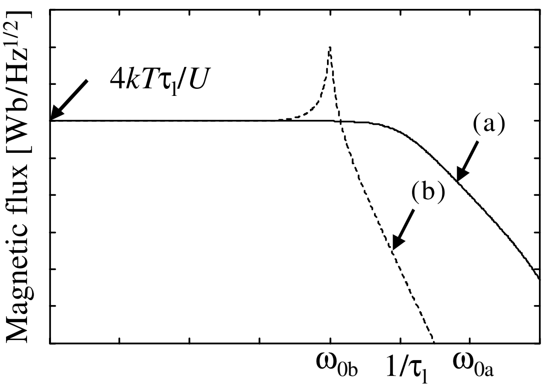

Figure 5. Spectral densities of the magnetic flux noise: (*a*) in the case $\omega_0 = \omega_{0a} > 1/\tau_1$, the noise has a knee at $\omega = 1/\tau_1$ and (*b*) in the case $\omega_0 = \omega_{0b} < 1/\tau_1$, the noise has a peak at $\omega = \omega_{0b}$ and decreases rapidly at $\omega > \omega_0$.

absorber. If we consider small signals and look at one Fourier component, we find that

$$\mathrm{i}\omega C T_\omega = -g T_\omega + P_\omega. \tag{18}$$

In case the temperature fluctuation, δT, is caused by a phonon noise, whose power spectral density is $(P_{\mathrm{phonon}})^2 = 4kT^2 g$, the spectral density of a temperature fluctuation is

$$(\delta T)^2 = \frac{(P_{\mathrm{phonon}})^2}{g^2 + \omega^2 C^2} = \frac{4kT^2}{g}\frac{1}{1+\omega^2\tau_{\mathrm{h}}^2}, \tag{19}$$

where $\tau_{\mathrm{h}} = C/g$ is the thermal relaxation time.

3.2. Johnson noise

The potential energy of SLJJ is given in equation (7). The stable points of Φ are where $U(\Phi)$ takes local minimum values. However, Φ fluctuates around the stable point because of a Johnson noise of normal state resistance in SLJJ. To estimate a fluctuation of Φ, we add a Johnson noise term I_{n} in equation (6). The fluctuation of Φ around the stable point is assumed to be small and we look at one Fourier component and find that

$$-U''(\Phi)\Phi_\omega = \mathrm{i}\omega\frac{1}{R_{\mathrm{n}}}\Phi_\omega - \omega^2 C_{\mathrm{j}}\Phi_\omega + I_{\mathrm{n}}. \tag{20}$$

The spectral density of a Johnson noise is written as $I_{\mathrm{n}}^2 = \frac{4kT}{R_{\mathrm{n}}}$ and the spectral density of magnetic flux fluctuation due to the Johnson noise is

$$(\delta\Phi_{\mathrm{L}})^2 = \frac{I_{\mathrm{n}}^2}{(\omega^2 C_{\mathrm{j}} - U'')^2 + \frac{\omega^2}{R_{\mathrm{n}}^2}} = 4kT\frac{1}{U''}\frac{\tau_1}{\left(\frac{\omega^2}{\omega_0^2}-1\right)^2 + \omega^2\tau_1^2}, \tag{21}$$

where $\omega_0 = \sqrt{\frac{U''}{C_{\mathrm{j}}}}$ is the resonant angular frequency and $1/\tau_1 = U''R_{\mathrm{n}}$ is the cut-off angular frequency. The spectral density of magnetic flux noise described in equation (21) has two types of forms according to the values of ω_0 and $1/\tau_1$, as shown in figure 5. In the case $\omega_0 = \omega_{0a} > 1/\tau_1$ as shown in figure 5(*a*), the noise has a knee at $\omega = 1/\tau_1$, and an approximation $\left(\frac{\omega^2}{\omega_0^2}-1\right)^2 + \omega^2\tau_1^2 \cong 1$ is satisfied at $\omega < 1/\tau_1$. In the case $\omega_0 = \omega_{0b} < 1/\tau_1$ as shown in figure 5(*b*), the noise has a peak at $\omega = \omega_{0b}$ and decreased rapidly at $\omega > \omega_{0b}$, and an approximation $\left(\frac{\omega^2}{\omega_0^2}-1\right)^2 + \omega^2\tau_1^2 \cong 1$ is satisfied at $\omega < \omega_{0b}$. In conclusion, an approximation $\left(\frac{\omega^2}{\omega_0^2}-1\right)^2 + \omega^2\tau_1^2 \cong 1$ is satisfied in the following range

$$\min(\omega_0, 1/\tau_1) > \omega. \tag{22}$$

3.3. Noise equivalent power

Noise equivalent power (NEP) is written as $\mathrm{NEP}^2 = S_\Phi\left(\frac{\delta\Phi}{\delta P}\right)^{-2}$, where S_Φ is the spectral density of the fluctuation of magnetic flux and $\left(\frac{\delta\Phi}{\delta P}\right)$ is the small signal responsivity as a function of frequency, which represents the relative amplitude of a modulation of Φ when the power into the absorber is modulated with an amplitude δP at a frequency f. The spectral density of fluctuation of magnetic flux, S_Φ, consists of the fluctuation due to the critical current, $\delta\Phi_I$, and the fluctuation due to Johnson noise in the SLJJ, $\delta\Phi_{\mathrm{L}}$. These two noise sources are not correlated to each other and can be added in quadrature. So, $S_\Phi = (\delta\Phi_I)^2 + (\delta\Phi_{\mathrm{L}})^2$ is described as

$$S_\Phi = \left(\frac{\partial\Phi}{\partial I_1}\right)^2\left(\frac{\mathrm{d}I_1}{\mathrm{d}T}\right)^2\frac{4kT^2}{g}\frac{1}{1+\omega^2\tau_{\mathrm{h}}^2} + \frac{4kT}{U''}\frac{\tau_1}{\left(\frac{\omega^2}{\omega_0^2}-1\right)^2+\omega^2\tau_1^2}. \tag{23}$$

Using equation (18), $\left(\frac{\delta\Phi}{\delta P}\right)$ is calculated to be

$$\left(\frac{\delta\Phi}{\delta P}\right)^2 = \left(\frac{\partial\Phi}{\partial I_1}\right)^2\left(\frac{\mathrm{d}I_1}{\mathrm{d}T}\right)^2\frac{1}{g^2}\frac{1}{1+\omega^2\tau_{\mathrm{h}}^2}. \tag{24}$$

After some calculation, NEP is described as

$$\mathrm{NEP}^2 = 4kT^2 g\left(1 + F\frac{\tau_1}{\tau_{\mathrm{h}}}\frac{1+\omega^2\tau_{\mathrm{h}}^2}{\left(\frac{\omega^2}{\omega_0^2}-1\right)^2+\omega^2\tau_1^2}\right), \tag{25}$$

where

$$F = \frac{2\pi a\cos\phi_1 + 1}{a^2\sin^2\phi_1}\frac{CTL}{\Phi_0^2}\frac{\xi^2}{x^2}. \tag{26}$$

Energy resolution can be calculated from the NEP as [7]

$$\Delta E = \left(\int_0^\infty \frac{4\,\mathrm{d}f}{\mathrm{NEP}^2(f)}\right)^{-1/2}, \tag{27}$$

where $f = \omega/2\pi$ is a frequency. When $1/\tau_{\mathrm{h}} \ll \min(\omega_0, 1/\tau_1)$ is satisfied, $\min(\omega_0, 1/\tau_1)$ can be regarded as infinity in calculating equation (27) and $\left(\frac{\omega^2}{\omega_0^2}-1\right)^2+\omega^2\tau_1^2 \cong 1$ is satisfied in the integral range as described in equation (22). Then, the integral can be solved easily and written as

$$\Delta E^2 \cong 4kT^2 C\sqrt{\left(1+F\frac{\tau_1}{\tau_{\mathrm{h}}}\right)\left(F\frac{\tau_1}{\tau_{\mathrm{h}}}\right)}. \tag{28}$$

In the case $F\frac{\tau_1}{\tau_{\mathrm{h}}} \ll 1$, the equation reduces to

$$\Delta E^2 \cong 4kT^2 C\sqrt{F\frac{\tau_1}{\tau_{\mathrm{h}}}}. \tag{29}$$

4. Discussions

In the previous section, we calculated the energy resolution of the microcalorimeter, which depends on F, τ_1, τ_h and (kT^2C), sometimes called a thermodynamic limit. In this section we estimate these parameters.

The thermal relaxation time $\tau_h = C/g$ can be estimated by using $C = \gamma T V \rho$ and $g = 5\Sigma V T^4$, and is written as $\tau_h = \frac{\gamma\rho}{5\Sigma T^3}$, where γ is Sommerfeld coefficient, V is the volume of an absorber, ρ is the density and Σ represents the intensity of an energy exchange rate between electron and phonon systems. In the case of using a Ag absorber, the parameters described above have the values of $\gamma = 6.02 \times 10^{-6}$ (J K^{-1} g^{-1}), $\rho = 10.5$ (g cm^{-3}) and $\Sigma = 2.1 \times 10^3$ (J s^{-1} K^{-5} cm^{-3}) [8], and τ_h is given as $6.0T^{-3}$ (ns). $\tau_1 = 33$ (ps) in the case $1/U'' = 10$ pH and $R_n = 300$ mΩ. In the case of an operating temperature of 100 mK, $\frac{\tau_1}{\tau_h}$ is $\frac{1}{1.8\times10^5}$.

We investigate the condition $1/\tau_h \ll \min(\omega_0, 1/\tau_1)$ in the case of an operating temperature of 100 mK, which ensures that equation (28) can be applied. The condition $1/\tau_1 \gg 1/\tau_h$ is satisfied, as previously discussed. A typical Josephson junction, whose thickness and relative dielectric constant of the insulator layer and junction area are 1 nm, 10 and 100 μm^2, has the value of C_j =10 pF. So, $\omega_0 = \sqrt{\frac{U''}{C_j}}$ becomes 10^{11} (Hz) and the condition $\omega_0 \gg 1/\tau_h$ is satisfied.

The parameter F given in equation (26) is complicated for optimization. Here, we estimate F for some typical cases from viewpoints of energy resolution and dynamic range. We take $\phi = \pi/2$ for simplicity; then, the dynamic range E_{dr} is approximately written as

$$E_{dr} = C\frac{dT}{dI_1}I_{1dr} = CT\frac{1}{8\pi^2a^2}\frac{\xi}{x}, \qquad (30)$$

the first factor of F, $\frac{2\pi a\cos\phi+1}{a^2\sin^2\phi}$, is given as $\frac{1}{a^2}$ and $1/U''$ equals L. Under the conditions of operating temperature = 100 mK, $C = 3.16\times10^{-12}$ (J K^{-1}) (corresponding to a $500\times500\times2$ μm^3 Ag absorber), $a = 1$ ($L = 10$ pH and $I_{10} = 0.2$ mA) and $\xi/x = 1/4$, we have the value of F as 45 000. In this condition, the dynamic range and the energy resolution are estimated to be 6.2 keV and 5.8 eV, respectively. When photons with less energy are measured, the dynamic range can become smaller and the energy resolution becomes better: with $\xi/x = 1/8$ and other parameters unchanged, the dynamic range and the energy resolution are obtained to be 3.1 keV and 4.2 eV, respectively.

Another way to improve the energy resolution of this Josephson junction microcalorimeter is to use a feedback method and enlarge its dynamic range. In a Josephson junction microcalorimeter, a feedback method is used to keep the magnetic flux in the loop constant. Two feedback methods are shown here as illustrated in figure 6.

The first one is to apply an additional external magnetic field according to the change of magnetic flux in the loop caused by the change of the critical current of the Josephson junction. When external magnetic flux exists, equation (2) is rewritten as

$$\Phi = Li + \Phi_x, \qquad (31)$$

where Φ_x is the external magnetic flux. Using this relation, equation (8) is rewritten as

$$I_1\sin\phi = \frac{1}{2}I_b - \frac{\Phi_0}{2\pi L}(\phi - \phi_x). \qquad (32)$$

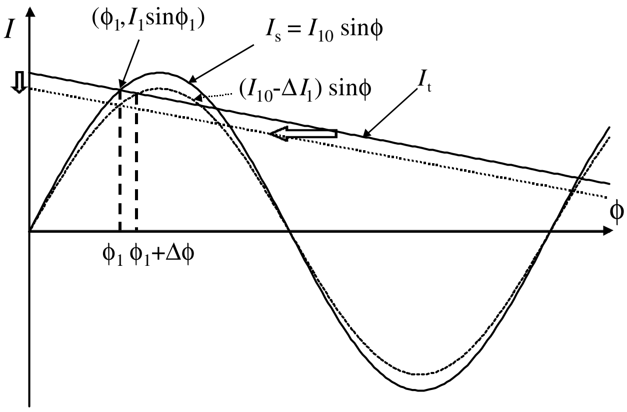

Figure 6. Feedback methods. In order to keep the magnetic flux in the loop constant when the critical current of the Josephson junction changes, (1) an additional external magnetic field is applied (a horizontal arrow) or (2) a bias current applied to the loop is changed (a vertical arrow).

In this way, by applying external magnetic flux, the I_t–ϕ line in figure 6 can be shifted horizontally and magnetic flux can be kept constant when the critical current of the Josephson junction is changed. The external magnetic flux required to keep the magnetic flux in the loop constant is

$$\Phi_x = -\left(2\pi a\cos 2\pi\frac{\Phi}{\Phi_0} + 1\right)\delta\Phi, \qquad (33)$$

where $\delta\Phi$ is the change of magnetic flux. Especially, when $\phi = \pi/2$ or $\Phi = \Phi_0/4$, Φ_x equals $-\delta\Phi$.

The second method is to change a bias current applied to the loop according to the change of the magnetic flux in the loop caused by the change of the critical current of the Josephson junction. Through this method, the I_t–ϕ line in figure 6 can be shifted vertically and magnetic flux in the loop can be kept constant. The required change of the bias current is written as

$$\delta I_b = -\frac{2}{L}\left(2\pi a\cos 2\pi\frac{\Phi}{\Phi_0} + 1\right)\delta\Phi, \qquad (34)$$

where $\delta\Phi$ is the change of magnetic flux. Especially, when $\phi = \pi/2$ or $\Phi = \Phi_0/4$, δI_b equals $-2\delta\Phi/L$.

5. Conclusion

Critical currents of Josephson junctions such as SNS and SNIS junctions, which include a normal metal barrier, can be controlled by changing the electron temperatures of the normal metal barriers. By coupling an SNS or an SNIS junction to a radiation absorber, it is possible to reduce the critical current of the junction in case of radiation incidence, and the system can be operated as a microcalorimeter. In order to measure the change of critical current of the Josephson junction, we suggested the magnetic calorimeter using a superconductor loop with a Josephson junction. In this microcalorimeter, magnetic flux in the superconductor loop is kept constant by a feedback method in a modulation of the critical current of the Josephson junction. The energy deposited by radiation is measured as a feedback signal. We estimated the energy resolution and the dynamic range of this Josephson junction microcalorimeter as 4.2 eV/5.8 eV and 3.1 keV/6.2 keV, respectively, with an Ag absorber of $500 \times 500 \times 2\ \mu m^3$ at the operating temperature of 100 mK. By using a feedback method, the dynamic range can be enlarged and the energy resolution can be improved.

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